

JUN 2014						
M	T	W	T	F	S	S
30	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29					

UNIT (V)

THE VIDYARTHI

May 2014

Homogeneous Linear D.E. with constant coefficients

wk 20 * day 132

Monday

12

Variable coefficients.

General form. $(x^n D^n + a_1 x^{n-1} D^{n-1} + a_2 x^{n-2} D^{n-2} + \dots + a_n) y = Q.$

where

$a_1, a_2, \dots, a_n = \text{constants}$

$Q = \text{Either const. or function of } (x)$

Solving Method:

Variable entering method.

put $x = e^z$, we get -

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

write

$$x \frac{dy}{dx} = \frac{dy}{dz}$$

$$\Rightarrow x \frac{d}{dx} = D$$

$$\text{or } \frac{dy}{dx} = \frac{1}{e^z} \frac{dy}{dz}$$

$$\text{i.e. } x D y = D y \Rightarrow x D = D$$

and

$$x^2 \frac{d^2 y}{dx^2} = D(D-1)y$$

Similarly $x^3 \frac{d^3 y}{dx^3} = D(D-1)(D-2)y$

$$x^n \frac{d^n y}{dx^n} = D(D-1)(D-2) \dots (D-n+1)y$$

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Ex: solve $x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} - 3y = x^2 + x$.

13

~~in homogeneous term as~~

14

put $x = e^z$ then.

$$[D(D-1)(D-2) + 2D(D-1) + 3D - 3]y = e^{2z} + e^z$$

15

$$\Rightarrow (D-1)(D^2+3)y = e^{2z} + e^z.$$

16

let's Auxillary eqⁿ is $(m-1)(m^2+3) = 0$

17

$$\Rightarrow m = 1 \pm \sqrt{3}i$$

18

$$\text{C.F. } (y) = c_1 e^z + [c_2 \cos \sqrt{3}z + c_3 \sin \sqrt{3}z]$$

$$\text{P.I.} = \frac{1}{f(D)} (e^{2z} + e^z)$$

Notes

$$= \frac{1}{(D-1)(D^2+3)} (e^{2z} + e^z)$$

Success is simply a matter of luck Ask any failure

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2014						
JUN	M	T	W	T	F	S
						1
30	2	3	4	5	6	7
1	8	9	10	11	12	13
2	14	15	16	17	18	19
3	20	21	22	23	24	25
4	26	27	28	29		

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May 2014

wk 20 • day 134

Wednesday

14

$$= \frac{1}{(D-1)(D^2+3)} e^{2z} + \frac{1}{(D-1)(D^2+3)} e^z$$

$$= \frac{e^{2z}}{(2-1)(2^2+3)} + \frac{e^z}{(2-1)(2^2+3)}$$

$$= \frac{1}{7} e^{2z} + \frac{1}{4} e^z$$

$$= \frac{1}{7} e^z + \frac{1}{4} e^z \frac{(1)}{(D+1)}$$

$$= \frac{1}{7} e^z + \frac{z}{4} e^z$$

$$\text{General sol.} = C_1 e^z + C_2 \cos \sqrt{3} z + C_3 \sin \sqrt{3} z + \frac{1}{7} e^{2z} + \frac{z}{4} e^z$$

16 put $x = e^z$

$\Rightarrow z = \log x.$ ✓

17
18
$$= c_1 x + c_2 \cos \sqrt{3}(\log x) + c_3 \sin \sqrt{3}(\log x)$$
$$+ \frac{1}{7} x^2 + \frac{2 \log x}{4}.$$

Notes

Part of success is preparation on purpose

**THE
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May 2014

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wk 20 • day 135

Thursday

MAY 2014						
M	T	W	T	F	S	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

2) $x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y = x^4$

3) $x^3 \left(\frac{d^3y}{dx^3} \right) + 2x^2 \frac{d^2y}{dx^2} + 2y = 10 \left[x + \frac{1}{x} \right]$

4) Solve $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} - 4y = x^4$

5) $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 4y = 2x^2$

2014

wk 20 • day 136

Friday

16

Solution

Sol. Put $x = e^z \Rightarrow z = \log x$

let us denote $\frac{d}{dz} = D$. then we can

write the given equation. is as

$$(D^2(D-1) - 4D + 6)y = x^4$$

$$\Rightarrow (D^2 - 5D + 6)y = x^4$$

its Auxillary equation is

$$m^2 - 5m + 6 = 0$$

$$(m-3)(m-2) = 0 \Rightarrow m = 3, 2.$$

Hence

$$C.F. (y) = C_1 e^{3x} + C_2 e^{2x}$$

$$= C_1 x^3 + C_2 x^2$$

17 Now P.I. = $\frac{1}{f(D)} x^4 = \frac{1}{f(D)} e^{4x}$

18 $= \frac{1}{(D-3)(D-2)} e^{4x}$

$= \frac{e^{4x}}{2}$

if $D=4$
then $f(D)$
 $= f(4) = 2 \neq 0$

Notes

$= \frac{x}{2}$

Hence Gen. Sol. = C.F. + P.I.

Motivation is the fuel, necessary to keep the human engine running

17

wk 20 • day 137

Saturday

12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

③. $x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 2y = 10 [x + yx]$

Sol. Put $x = e^z \Rightarrow z = \log x$.
and write $\frac{d}{dx} = D$. then we get.

$$D(D-1)(D-2) + 2D(D-1) + 2 = 10 [e^z + e^{-z}]$$

$\Rightarrow$ Let's Auxiliary eq. is

$$m(m-1)(m-2) + 2m(m-1) + 2 = 0.$$

$$m(m-1)(m-2+2) + 2 = 0$$

15

$$m^3 - m^2 + 2 = 0$$

$$\Rightarrow (m+1)(m^2 - 2m + 2) = 0 \Rightarrow m = -1, (1 \pm i)$$

16

$$\text{Hence C.f. (y)} = c_1 e^{-x} + e^x (c_2 \cos x + c_3 \sin x)$$

17

$$= c_1 x + x c_2 \cos(\log x) + c_3 x \sin(\log x)$$

18

$$\text{P.I.} = \frac{1}{f(D)} 10 [e^x + e^{-x}]$$

18 Sunday

$$= \frac{1}{(D^3 - D^2 + 2)} 10 [e^x + e^{-x}]$$

Notes

$$= 10 \left[\frac{e^x}{(D^3 - D^2 + 2)} + \frac{e^{-x}}{(D^3 - D^2 + 2)} \right]$$

You have achieved success if you have lived well, laughed often and loved much

10e² taking first part

$$= 10 \frac{e^2}{D^3 - D^2 + 2}$$

$$= \frac{10e^2}{1-1+2} = \frac{10e^2}{2}$$

$$\text{II part} = \frac{10e^{-2}}{(D^3 - D^2 + 2)}$$

$$= 10e^{-2} \frac{1}{(D-1)^3 - (D-1)^2 + 2}$$

$$= 10e^{-2} \frac{1}{(D-1)^2((D-1)^2 - 1) + 2} \quad (1)$$

$$= 10e^{-2} \frac{1}{(D-1)(D^2 + 1 - 2D + 1) + 2}$$

$$= 10e^{-2} \frac{1}{(D-1)(D^2 - 2D + 2)}$$

$$= 10e^{-2} \frac{1}{D(D-1)(D-2) + 2}$$

$$= 10e^{-2} \frac{1}{D(D^2 - 3D + 2) + 2} = 10e^{-2} \frac{1}{(D^3 - 3D^2 + 2D + 2)}$$

$$= 10e^{-2} \frac{1}{2(1 + \frac{D^3 - 3D^2 + 2D}{2})} \quad (1)$$

Success is still the constant application of the Golden Rule

Solve it