THE
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6319

B. A./B. Sc. (Third Semester)
EXAMINATION, 2017-18
MATHEMATICS
Paper Second
(Differential Equation)

Time : Three Hours] [*Maximum Marks : 75*

Note : There are three Sections A, B and C in this paper. Section A is compulsory. Attempt any *four* questions from Section B and any *two* questions from Section C. Marks allotted to each question are indicated against them. Answer the questions in serial order as far as possible.

Section—A $1\frac{1}{2}$ each

1. (a) Order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 - 2\frac{dy}{dx} + 6y = x^5$ are respectively :

(i) 3, 2

(ii) 2, 3

(iii) 2, 5

(iv) 3, 5

(b) The number of arbitrary constants in the particular solution of a differential equation of second order is :

(i) 0

(ii) 1

(iii) 2

(iv) 3

(c) The number of arbitrary constants in the general solution of the differential equation of order four is :

(i) 1

(ii) 2

(iii) 3

(iv) 4

(d) Which differential equation has $y = C_1e^x + C_2e^{-x}$ as the general solution ?

(i) $\frac{d^2y}{dx^2} + y = 0$

(ii) $\frac{d^2y}{dx^2} - 1 = 0$

(iii) $\frac{d^2y}{dx^2} - y = 0$

(iv) $\frac{d^2y}{dx^2} + 1 = 0$

(e) Solution of the differential equation $\frac{dy}{dx} = \frac{1}{x}\cot y$ is :

(i) $x \cos y = C$

(ii) $y \cos x = C$

(iii) $y = \cos x$

(iv) $y = \log x . \cos x$

(f) Integrating factor of the differential equation $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$ is :

(i) y^2

(ii) $\frac{1}{y}$

(iii) y

(iv) y^3

(g) The differential equation of the form $y = x.F(p) + f(p)$ is known as :

(i) Lagrange's equation

(ii) Clairaut's equation

(iii) Total differential equation

(iv) Homogeneous equation

(h) To find the singular solution of a differential equation, we use :

(i) c-discriminant relation

(ii) p-discriminant relation

(iii) Both (i) and (ii)

(iv) None of these

(i) A solution, which cannot be obtained from general solution by assigning some particular value of arbitrary constant is known as :

(i) Particular solution

(ii) Singular solution

(iii) Integrating factor

(iv) Complementary function

(j) The complete primitive can be obtained by :

(i) P. I.

(ii) C. F.

(iii) C. F. + P. I.

(iv) None of these

Section—B

2. (a) Find the differential equation of the family of curves $y = Ae^{2x} + Be^{-2x}$, where A and B are parameters. 4

(b) Solve : $3\frac{1}{2}$

$$\frac{dy}{dx} = (4x + y + 1)^2$$

3. Solve : $7\frac{1}{2}$

$$xyp^2 - (x^2 + y^2)p + xy = 0$$

4. Find the orthogonal trajectories of $x^2 + y^2 = 2ax$. $7\frac{1}{2}$

5. (a) Solve : $3\frac{1}{2}$

$$(D^3 + 6D^2 + 11D + 6)y = 0$$

(b) Solve : 4

$$(D^2 - 2D + 4)y = e^x \cos x$$

6. Solve : $7\frac{1}{2}$

$$x^2\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+6y=x$$

7. (a) Solve : $3\frac{1}{2}$

$$\frac{dx}{dt}=-wy$$

$$\frac{dy}{dt}=wx$$

(b) Solve : 4

$$\frac{dx}{y^2+z^2-x^2}=\frac{dy}{-2xy}=\frac{dz}{-2xz}$$

8. (a) Show that the curve in which the angle between the tangent and the radius vector at any point is half of the vector angle is a cardioid. 4

(b) Solve : $3\frac{1}{2}$

$$yz\log z\,dx-zx\log z\,dy+xy\,dz=0$$

9. Solve :

$$\frac{d^2y}{dx^2}-2\tan x\frac{dy}{dx}+5y=0$$

Section—C

10. (a) Solve : $7\frac{1}{2}$

$$(1+x^2)\frac{dy}{dx}+2xy=\cos x$$

(b) Find the singular solution of $y=px+\sqrt{b^2+a^2p^2}$. $7\frac{1}{2}$

11. (a) Solve : $7\frac{1}{2}$

$$(y^2+2x^2y)dx+2\left(x^3-\frac{xy}{2}\right)dy=0$$

(b) Solve : $7\frac{1}{2}$

$$x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+y=\frac{dx}{(1-x)^2}$$

12. (a) Solve : $7\frac{1}{2}$

$$(D^2-3D+2)y=e^{5x}$$

(b) Determine C_1 and C_2 so that $y(X)=C_1e^{2E}+C_2e^{X}+2\sin X$ will satisfy the conditions $y(0)=0$ and $y'(0)=1$. $7\frac{1}{2}$

13. (a) Solve : $7\frac{1}{2}$

$$(2x+y+3)dx=(2y+x+1)dy$$

(b) Solve : $7\frac{1}{2}$

$$(D^3+3D^2+3D+1)y=e^{-x}$$